NAME (Please print) _____ _____
(Last) (First)

MIT ID # _____ E-Mail _____

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

MATH DIAGNOSTIC FOR PHYSICS PLACEMENT

The purpose of this diagnostic test is to determine your fluency with high school mathematics. The Physics Department will use your score to determine which version of first-year physics you will be allowed to take.

The Math Diagnostic is not intended to test your ability to figure out forgotten things from scratch. If a question seems to require a formula which you have forgotten, don't spend an excessive amount of time on it; just go on to another question. The test is a mixture of easy and hard questions. You should not worry if you cannot answer all the questions.

Directions

- No calculators, books, or other tools may be used during this test.
- Fill in the information above and write your name at the top of each page.
- There are twenty questions, five in each section.
- Do all your work on these sheets in the space provided, using the reverse side if necessary.
- Box your answers. Show all your work; partial credit will be given.
- Check over your work for possible errors.
- Stop working when the proctor calls time (after 90 minutes).

Do not write below this line.

A1	B1	C1	D1
A2	B2	C2	D2
A3	B3	C3	D3
A4	B4	C4	D4
A5	B5	C5	D5
A total:	B total:	C total:	D total:
Grader:	Grader:	Grader:	Grader:

NAME _____

Math Diagnostic, Part A

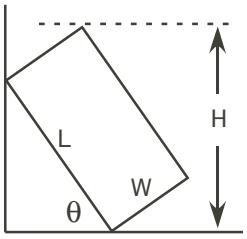
A1. Express in scientific notation (that is $a \times 10^k$ where $1 \le a < 10$) the expression

$$\frac{18 \times 10^{-6}}{(.003)(12 \times 10^{-2})} = \frac{1.8 \times 10^{-5}}{3 \times 10^{-3} \times 1.2 \times 10^{-1}} = \frac{0.6}{1.2} \times 10^{-1} = \boxed{5 \times 10^{-2}}$$

A2. Expand $(1 - ax)^3 = \boxed{1 - 3ax + 3a^2x^2 - a^3x^3}$ using the binomial expansion.

Also $= (1 - 2ax + a^2x^2)(1 - ax) = 1 - 3ax + 3a^2x^2 - a^3x^3$ by direct multiplication.

A3. A rectangular box of length L and width W rests against a wall, making an angle θ with respect to the floor. What is the height H of the top edge of the box above the floor?



$\boxed{H = L \sin\theta + W \cos\theta}$

A4. Find the derivative with respect to x of $f(x) = \sin \dfrac{1}{\sqrt{x}}$.

$$\frac{d}{dx} \sin x^{-1/2} = (\cos x^{-1/2})(-\tfrac{1}{2}\, x^{-3/2}) = \boxed{-\tfrac{1}{2}\, x^{-3/2} \cos x^{-1/2}}$$

A5. Britney has 15 coins, all nickels and dimes. If the nickels were changed to quarters and the dimes changed to nickels, the total value of the coins would remain unchanged. How many nickels and how many dimes does she have?

n+d=15

$$\begin{aligned} 5n + 10d &= 25n + 5d \\ 5d &= 20n \\ d &= 4n \end{aligned}$$

$n + 4n = 15$

$\boxed{n = 3}$ $\boxed{d = 12}$

NAME _____

Math Diagnostic, Part B

B1. Express in the form $a + bi$ where $i \equiv \sqrt{-1}$:

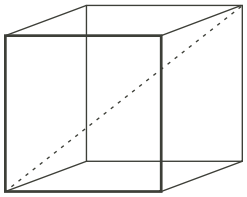
a) $\dfrac{3+i}{1-i} = \dfrac{(3+i)(1+i)}{(1-i)(1+i)} = \dfrac{3-1+4i}{1+1} = \boxed{1+2i}$

b) $e^{-ix} = \underline{\cos x - i \sin x}$

B2. Find all possible values of x which solve the equation $x^4 - 13x^2 + 36 = 0$.

$x^4 - 13x^2 + 36 = (x^2-4)(x^2-9) = 0 \Rightarrow \boxed{x = \pm 2, \pm 3}$

B3. In the figure of a cube below, the dashed line indicates the body diagonal. If the cube has sides of length a, what is the length of the body diagonal?

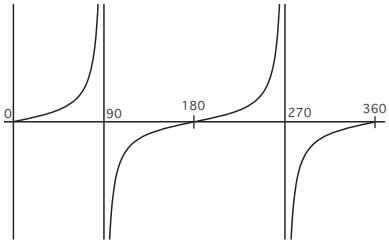


By vectors: $\vec{d} = a\hat{x} + a\hat{y} + a\hat{z} \Rightarrow |\vec{d}| = \boxed{\sqrt{3}\,a}$

By right triangles: Face diagonal $= a\sqrt{1^2+1^2} = \sqrt{2}\,a$

Body diagonal $= a\sqrt{(\sqrt{2})^2+1^2} = \sqrt{3}\,a$

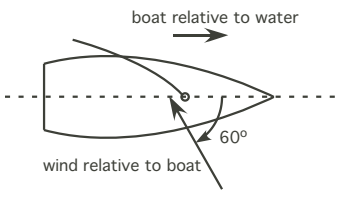
B4. Sketch the function $\tan(\theta)$ for the region $0 \le \theta < 360^o$. What is the corresponding range of the argument in radians?



$0 \le \theta < 360^o$ in degrees is equivalent to

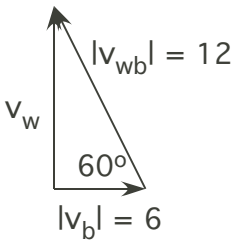
$\boxed{0 \le \theta < 2\pi \text{ in radians.}}$

B5. A sailboat is moving forward at a speed of 6 knots. A gauge attached to its mast indicates a wind speed of 12 knots at an angle of 60 degrees, both relative to the boat. What is the wind speed relative to the water?



$\vec{v}_w = \vec{v}_{wb} + \vec{v}_b$

$\boxed{|\vec{v}_w| = 6\sqrt{3} \text{ knots}}$



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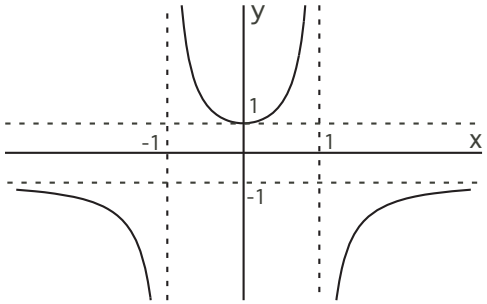
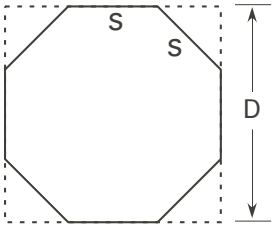
Math Diagnostic, Part C

C1. Sketch $y(x) = \dfrac{1+x^2}{1-x^2}$ in the $x-y$ plane.



C2. A regular octagon is formed by cutting 45 degree right triangles off of the corners of a square of side D. What is the area of the resulting octagon?

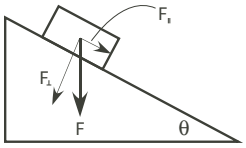


$s + 2(s/\sqrt{2})\,x = (1+\sqrt{2})s = D \;\Rightarrow\; s = D/(1+\sqrt{2})$

$A = D^2 - 2(s/\sqrt{2})^2 = D^2 - s^2 = D^2\,(1 - (\frac{1}{1+\sqrt{2}})^2)$

$= \boxed{(\frac{2}{1+\sqrt{2}})\,D^2}$ equivalently, by a different method $(\frac{2+2\sqrt{2}}{3+2\sqrt{2}})\,D^2$

C3. A gravitational force of magnitude F acts vertically downward on a block resting on a plane. The plane makes an angle of θ with the horizontal. What is the magnitude of the component of the gravitational force acting parallel to the plane?



$\boxed{|F_{\parallel}| = F\sin\theta}$

C4. Find the first and second derivatives with respect to x of $f(x) = \dfrac{e^{x/a} - e^{-x/a}}{2}$.

$$\frac{df}{dx} = \boxed{\tfrac{1}{2a}\,(e^{x/a} + e^{-x/a})}$$

$$\frac{d^2f}{dx^2} = \boxed{\tfrac{1}{2a^2}\,(e^{x/a} - e^{-x/a})}$$

C5. A party of fishermen rented a boat for \$240. Two of the fishermen had to withdraw from the party and, as a result, the share of each of the others was increased by \$10. How many were in the original party?

$$\frac{240}{n} + 10 = \frac{240}{n-2}$$

$$\begin{aligned} 240(n-2) + 10n(n-2) &= 240n \\ -480 + 10n^2 - 20 &= 0 \\ n^2 - 2n - 48 = (n-8)(n+6) &= 0 \end{aligned}$$

$$\Rightarrow \boxed{n = 8}$$

NAME _____

Math Diagnostic, Part D

D1. Solve for x.

$$\frac{2^x}{16} = (2^x)^3$$

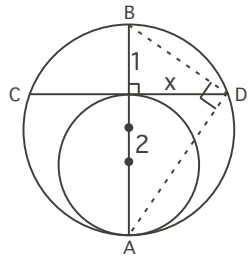
$$\begin{aligned} \tfrac{1}{16} &= (2^x)^2 \\ \tfrac{1}{4} &= 2^x = (\tfrac{1}{2})^{-2} \\ &\Rightarrow \boxed{x = -2} \end{aligned}$$

D2. Solve for x.

$$\frac{14}{x^2-9} - \frac{7}{x+3} = \frac{1}{x-3}$$

$$\begin{aligned} 14 - 7(x-3) &= x+3 \\ -7x + 35 &= x+3 \\ 32 &= 8x \quad \Rightarrow \quad \boxed{x = 4} \end{aligned}$$

D3. Two circles with diameters 2 and 3 centimeters are tangent at A. The line AB goes through the centers of both circles. The line CD is perpendicular to AB and tangent to the smaller circle. What is the length of the line CD?



Use similar triangles to see that

$$\frac{1}{x} = \frac{x}{2} \quad \Rightarrow \quad x = \sqrt{2}$$

$$\boxed{CD = 2\sqrt{2}}$$

D4. Evaluate the definite integral $\displaystyle\int_1^e \frac{1}{x} = \Big[\ln x = \ln e - \ln 1 = 1 - 0 = \boxed{1}$.

D5. A truck and a sports car, each at its own constant speed, travel the 500 miles from Chicago to Memphis. The sports car leaves Chicago 1 hour after the truck then passes it 2 hours later. After arriving in Memphis the sports car waits for 1 hour, then begins the return journey. After covering a distance of 40 miles, it meets the incoming truck. Find the speeds of the truck and sports car.

distance to first passing $= 3v_t = 2v_c \quad \Rightarrow v_c = 3/2 v_t$

$$\begin{aligned} \text{time to second passing} = \frac{500-40}{v_t} &= \frac{500+40}{v_c} + 2 \\ \frac{460}{v_t} &= \frac{2(540)}{3v_t} + 2 = \frac{360}{v_t} + 2 \\ \frac{100}{v_t} &= 2 \quad \Rightarrow \quad \boxed{v_t = 50} \quad \rightarrow \boxed{v_c = 75 \text{ miles per hour}} \end{aligned}$$